Difference Between the Effectiveness-NTU and LMTD Methods

What is the difference between the Effectiveness-NTU and LMTD Methods for analyzing heat exchangers?

By Jeff Sines, Senior Product Engineer, Engineered Software, Inc.

As with any engineering problem, there are various ways to approach a solution when sizing and selecting a heat exchanger or analyzing its thermal performance. If the selected heat exchanger is undersized, the design heat transfer conditions will not be achieved. Resulting in less heat transfer and higher outlet fluid temperatures, which leads to off-quality production, exceeding environmental limits, or creating safety hazards that require mitigation. Corrective action would require the purchase and installation of a properly sized heat exchanger, causing additional downtime for installation.

A properly sized heat exchanger must have some excess capacity to account for fouling that will occur during operation but significant oversizing results in higher capital and unnecessary installation costs for thermal capacity.

The thermal capacity of a heat exchanger is its ability to transfer heat between two fluids at different temperatures. It is a function of the heat exchanger design and the fluid properties on both sides. The thermal capacity of the heat exchanger will match the thermal capacity required by the process conditions (temperatures and flow rates) if it has sufficient heat transfer area to do so.

The Effectiveness-NTU and Log Mean Temperature Difference (LMTD) are two solution methods that approach heat exchanger analysis from different angles. Both methods share common parameters and concepts and will arrive at the same solution to heat exchanger thermal capacity. To understand the difference between these two methods, we need to understand the key terminology and the equations used in each solution method.

Starting in PIPE-FLO® Professional 15.0, all parameters in both methods are calculated to help system design engineers, heat exchanger manufacturers, and plant engineers size, select, and evaluate the performance of heat exchangers in their piping systems.

The LMTD Method

The LMTD method is perhaps the most commonly known method used to analyze heat transfer in heat exchangers and is described in the Tubular Exchanger Manufacturers Association (TEMA) Standards and other well-known industry references. The equation to calculate the heat transfer rate is given by:

\[ \dot{Q} = (UA)(CMTD) = (UA)(CF)(LMTD) \]

Where:
• \( Q \) = Heat Transfer Rate (BTU/hr or W)
• \( UA \) = Heat Exchanger Thermal Capacity (BTU/hr·°F or W/°C)
• \( U \) = Heat Transfer Coefficient (BTU/hr·ft\(^2\)·°F or W/m\(^2\)·°C)
• \( A \) = Heat Transfer Area (ft\(^2\) or m\(^2\))
• \( \text{CMTD} \) = Corrected (or True) Mean Temperature Difference (°F or °C)
• \( \text{LMTD} \) = Logarithmic Mean Temperature Difference (°F or °C)
• \( \text{CF} \) = Configuration Correction Factor (dimensionless)

Log Mean Temperature Difference (LMTD)

The Log Mean Temperature Difference (LMTD) is calculated using the equation for the counter current/flow pattern (unless it is a completely single path parallel flow pattern):

\[
\text{LMTD} = \frac{dT_A - dT_B}{\ln\left(\frac{dT_A}{dT_B}\right)}
\]

Where:

• \( dT_A = (T_{\text{hot in}} - T_{\text{cold out}}) \)
• \( dT_B = (T_{\text{hot out}} - T_{\text{cold in}}) \)

Configuration Correction Factor (CF)

The Configuration Correction Factor (CF) accounts for the deviation of the internal flow pattern of the actual heat exchanger from that of a single pass counter current flow pattern. Some manufacturers provide a CF data table for their heat exchanger while others determine CF using a standard graph from the Tubular Exchanger Manufacturers Association (TEMA) for the actual heat exchanger configuration.

To determine the CF, two temperature difference ratios (\( P \) and \( R \)) must first be calculated from the four fluid temperatures entering and leaving the heat exchanger.

Temperature Effectiveness (P)

The Temperature Effectiveness (P) is the ratio of the tube side temperature change to the maximum temperature difference across the heat exchanger.

\[
P = \frac{dT_{\text{tube}}}{dT_{\text{max}}}
\]

Where:

• \( dT_{\text{max}} = (T_{\text{hot in}} - T_{\text{cold in}}) \)

Temperature Difference Ratio (R)

The Temperature Difference Ratio (R) is the ratio of the temperature change across the shell side to the temperature difference across the tube side.

\[
R = \frac{dT_{\text{shell}}}{dT_{\text{tube}}}
\]

\( P \) is limited to values between 0 and 1.0, but the \( R \)-value can be greater than 1.0 because the tube side is used as the reference side.

Corrected Mean Temperature Difference (CMTD)
After calculating P and R, CF is then determined graphically using the location of the P value on the appropriate R curve. In other words, the heat exchanger operates at a point on an R Curve based on the Temperature Effectiveness established by the operating conditions. The location of the operating point establishes the Configuration Correction Factor that is used to calculate the Corrected (or true) Mean Temperature Difference across the heat exchanger.

\[ CMTD = (CF)(LMTD) \]

It is desirable to operate a heat exchanger within a reasonable range around the “knee” of the R curves.

**Required Thermal Capacity (UA) by LMTD Method**

The required thermal capacity (UA) needed to achieve the heat transfer rate established by the temperatures and flow rates is calculated from the Heat Transfer Rate and the Corrected Mean Temperature Difference.

\[ U_{A\text{required}} = \frac{\dot{Q}}{(CMTD)} \]

The heat exchanger will operate at this thermal capacity as long as it has sufficient heat transfer area at these operating conditions, including a factor for fouling.

**Effectiveness-NTU Method**

The Effectiveness-NTU method takes a different approach to solving heat exchange analysis by using three dimensionless parameters: Heat Capacity Rate Ratio (HCRR), Effectiveness (), and Number of Transfer Units (NTU). The relationship between these three parameters depends on the type of heat exchanger and the internal flow pattern.

**Heat Capacity Rate Ratio (HCRR)**

The first dimensionless parameter is the Heat Capacity Rate Ratio (HCRR), the ratio of the minimum to the maximum value of Heat Capacity Rate (HCR) for the hot and cold fluids. The HCR of a fluid is a measure of its ability to release or absorb heat. The HCR is calculated for both fluids as the product of the mass flow rate times the specific heat capacity of the fluid.

\[ HCR = wc_p \]

Where:

- \( HCR \) = Heat Capacity Rate of the hot or cold fluid (BTU/hr·°F or W/°C)
- \( w \) = mass flow rate of the fluid (lb/hr or kg/sec)
- \( c_p \) = specific heat capacity of the fluid (BTU/lb·°F or J/kg °C)

\[ HCR = \frac{HCR_{\text{min}}}{HCR_{\text{max}}} \]

Where:

- \( HCR_{\text{min}} \) = minimum value of Heat Capacity Rate of the hot or cold fluid
- \( HCR_{\text{max}} \) = maximum value of Heat Capacity Rate of the hot or cold fluid

The HCRR is limited to values between 0 and 1.0 and is similar to the R ratio in the LMTD method. When \( R > 1 \), \( HCRR = 1/R \).

**Effectiveness ()**

The second parameter, Effectiveness (), is defined as the ratio of the actual heat transfer rate to the maximum possible heat transfer rate for the given flow and temperature conditions.

\[ \epsilon = \frac{\dot{Q}}{\dot{Q}_{\text{max}}} \]

Where:

- \( \dot{Q} \) = actual heat transfer rate
- \( \dot{Q}_{\text{max}} \) = maximum possible heat transfer rate

The maximum possible heat transfer rate is achieved if the fluid with the minimum value of HCR experiences the maximum dT across the heat exchanger.

\[ \dot{Q}_{\text{max}} = (HCR_{\text{min}})(dT_{\text{max}}) \]
Where:

- \( HCR_{\text{min}} \) = minimum value of heat capacity rate between the hot and cold fluids
- \( dT_{\text{max}} = (T_{\text{hot in}} - T_{\text{cold in}}) \)

This definition of Effectiveness (\( \eta \)) is similar to the definition of Temperature Effectiveness (\( P \)) in the LMTD method but uses the side with the minimum value of the Heat Capacity Rate as the reference instead of the tube side. When \( R \leq 1.0 \), \( \eta = P \). For values of \( R > 1.0 \), \( \eta = PR \).

**Number of Transfer Units (NTU)**

The last dimensionless parameter, the Number of Transfer Units (NTU), is the ratio of the heat exchanger’s ability to transfer heat (\( UA \)) to the fluid’s minimum ability to absorb heat (\( HCR_{\text{min}} \)).

\[
NTU = \frac{UA}{HCR_{\text{min}}}
\]

The NTU is a function of the Effectiveness and HCRR established by the process temperatures and flow rates and is indicative of the size of the heat exchanger needed. The greater the value of NTU, the larger the heat transfer surface area (\( A \)) required to meet the process conditions.

NTU is normally not calculated from the equation above, but instead solved graphically or using equations for NTU as a function of the Effectiveness and HCRR.

**Required Thermal Capacity (UA) by -NTU Method**

The thermal capacity (UA) required to achieve the heat transfer rate is determined by re-arranging the NTU equation after determining the value of NTU for the particular heat exchanger configuration.

\[
UA_{\text{required}} = (NTU)(HCR_{\text{min}})
\]

Similar to the LMTD method, the heat exchanger will operate at this thermal capacity as long as it has sufficient heat transfer area at these operating conditions, taking into account the fouling factor.

**Mathematical NTU Solution**

Equations exist for calculating NTU from Effectiveness and HCRR for some, but not all, heat exchanger configurations. For example, for a pure single pass counter current flow heat exchanger:

\[
NTU = \left( \frac{1}{HCRR - 1} \right) \ln \left( \frac{\epsilon - 1}{HCRR \cdot \epsilon - 1} \right)
\]

Equations for NTU vary by heat exchanger configuration, but the mathematical relationship for some types of heat exchangers is not readily available or easily derived.

**Graphical NTU Solution**

As with the LMTD method, a solution can be found graphically using an -NTU curve for the actual heat exchanger configuration. NTU is determined using the location of the Effectiveness on the appropriate HCRR curve.

The -NTU curves for pure single pass parallel flow (worst configuration) and pure single pass counter current flow (best configuration) are shown. The counter current configuration shows all HCRR lines approaching an Effectiveness of 1.0 as NTU increases, indicating that this configuration can achieve any value of Effectiveness for all ranges of HCRR, as long as the heat transfer area is sufficiently large enough.

However, the parallel flow configuration shows an achievable Maximum Effectiveness for each HCRR curve. Since the heat exchanger must operate on an HCRR curve within the region defined by the HCRR = 0 and HCRR = 1.0 curves, values of Effectiveness above the Maximum Effectiveness cannot be achieved regardless of how much heat transfer surface area is available (i.e. how large the NTU value becomes).
Comparing LMTD and -NTU Curves

The -NTU curves for two types of shell and tube heat exchangers is shown. The left curve shows a one shell / two tube pass heat exchanger and the right curve shows a two shell / four tube pass heat exchanger. These heat exchangers have corresponding CF-P-R curves shown in the discussion of the LMTD method.

The one shell / two tube pass heat exchanger has some portion of flow that is counter flow, some is parallel flow, and some is cross flow. Each HCRR curve flattens to a maximum value of Effectiveness as was the case for the pure single pass parallel flow heat exchanger. For this configuration, the Maximum Effectiveness for a given HCRR curve is greater than that for a pure single pass parallel flow configuration.

The two shell / four tube pass heat exchanger -NTU curve shows that the more shells and tube passes in the heat exchanger, the more the performance approaches that of a single pass counter current heat exchanger.

Engineering Analogies

Analogies are often made between concepts in many engineering disciplines. Voltage drop, current, and electrical resistance are analogous to pressure drop, fluid flow, and hydraulic resistance, which are analogous to the temperature difference, heat transfer rate, and thermal resistance. Similarly, a direct comparison can be made between the thermal capacity of a heat exchanger and the flow capacity of a control valve.

A control valve is sized and selected to meet the hydraulic requirements of the piping system, which includes the design flow rate and pressure drop across the valve. The control valve is slightly over-sized to ensure sufficient capacity to deliver the required flow. The Flow Coefficient (Cv) represents the flow capacity of a valve, which varies with valve position from zero at fully closed to a maximum value at 100% open. Between 0% and 100% the valve position is throttled such that the flow coefficient of the valve meets the flow coefficient required by the process conditions (flow rate and pressure drop).

Similarly, a heat exchanger is sized and selected to meet the thermal requirements of the system, which includes the design heat transfer rate at a true mean temperature difference across the heat exchanger. The Thermal Capacity (UA) represents the heat exchanger’s ability to transfer heat and has a maximum value based on the heat transfer surface area (A) and the maximum possible value of the heat transfer coefficient (U); which depends on both fluids’ convection heat transfer coefficients, tube wall thickness, material conduction heat transfer coefficient, and fouling factors. The heat exchanger is not always operating at this maximum Thermal Capacity but instead can be “throttled” to meet the Thermal Capacity required by the process conditions. This throttling of Thermal Capacity is accomplished by changing both fluids’ convection heat transfer coefficients by regulating the flow rates (and, therefore, the outlet temperatures) with control valves.

Summary

Piping systems are built to transport fluid to do work, transfer heat, and make a product. When designing piping systems to support heat transfer between fluids, both the hydraulic and thermal conditions must be evaluated to ensure the proper equipment is selected and installed. Evaluating both the hydraulic and thermal conditions of a system can be a daunting task for any engineer and is often divided into different groups who specialize in a specific field. The division often results in misunderstanding, miscommunication, and mistakes when integrating the work of the various groups.

Improperly sized equipment, whether the equipment is a pump, control valve or heat exchanger, results in additional capital and maintenance costs, off-quality production, environmental excursions, and potentially increase safety risks. Using comprehensive software tools like PIPE-FLO® Professional helps the design engineer, process engineer, and owner/operator have a clear view of the system operation.