Relationship Between Flow Coefficient and Discharge Coefficient

What is the relationship between the Flow Coefficient ($C_v$) and the Discharge Coefficient ($C_d$) ?

Good communication between various groups involved with fluid piping systems is critical for the proper design, operation, and determination of cost for many systems in residential, commercial, and industrial applications. It is crucial that the engineer understand and apply equations correctly to prevent costly mistakes in the sizing and selection of equipment, operating within safety limits, and avoiding unnecessary modifications later in plant life. One potential area for costly miscommunication is the use of coefficients for devices that have a fluid flowing through them. Manufacturers of various equipment use different coefficients to characterize the hydraulic performance of their devices, and these differences must be understood when applying them to calculations involving piping systems.

In a previous article, the difference between the Resistance Coefficient ($K$) and Flow Coefficient ($C_v$) was evaluated and a relationship between the two was derived. The Flow Coefficient ($C_v$ in US units, $K_v$ in SI units) is typically associated with the hydraulic performance of a control valve, but other devices such as safety relief valves are characterized by the Discharge Coefficient ($C_d$, sometimes designated by $K_d$), which is also associated with orifices and nozzles. They are not numerically equivalent, so what is the relationship between the two?

There are various standards in the U.S. and internationally that are used to size and select control valves and relief valves, most notably the ANSI/ISA-75.01.01 Flow Equations for Sizing Control Valves (IEC 60534-2-1 equivalent) and the API Standard 520 Part 1, Sizing, Selection, and Installation of Pressure-relieving Devices in Refineries. These two standards can be used to derive the relationship between the Flow Coefficient ($C_v$) and the Discharge Coefficient ($C_d$) for relief valves. There are minor differences in the nomenclature used in each standard, so for the purpose of this article, the nomenclature will be defined for the equations below along with the engineering units being used.

Control Valve Sizing Equations

When sizing a control valve, the minimum required flow coefficient is calculated based on the design flow rate and expected pressure drop across the valve, and a valve is selected that has a flow coefficient greater than the calculated value. Here's the general sizing equation for control valves for incompressible fluids according to ANSI/ISA-75.01.01 Equation 1, non-choked turbulent flow:

$$C_v = \frac{Q}{N_1} \sqrt{\frac{\rho_1}{dP}}$$

Where:

- $Q$ = volumetric flow rate (gpm, m³/hr, or lpm)
- $dP$ = pressure drop across the valve (psi, kPa, or bar)
- $\gamma$ = density of the fluid flowing through the valve (lb/ft³ or kg/m³)
- $\rho_0$ = density of water flowing through the valve (lb/ft³ or kg/m³)
- $SG = \gamma/\rho_0$ = specific gravity of the fluid (dimensionless)
- $N_1$ = constant that depends on the units used for $Q$ and $dP$ ($N_1 = 1.0$ for units of gpm and psi)
There are other factors that may be included in the sizing equation to account for piping geometry, high viscosity, or choked flow conditions. Using U.S. units of gpm and psi, the flow coefficient equation in its simplest form is:

$$ C_v = Q \sqrt{\frac{SG}{dP}} $$

(2)

**Relief Valve Sizing Equations**

When sizing a relief valve, the minimum required effective area is calculated and a relief valve is selected that has an effective area greater than the calculated value. The sizing equation for relief valves for liquids using U.S. units according to Equation 28 in the API 520 standard is:

$$ A = \frac{Q}{38K_dK_wK_cK_v} \sqrt{\frac{SG}{P_1 - P_2}} $$

(3)

Where:

- $A$ = required effective orifice area (in$^2$)
- $Q$ = volumetric flow rate (gpm)
- $SG$ = specific gravity of the fluid (dimensionless)
- $P_1$ = upstream relieving pressure (psig)
- $P_2$ = backpressure (psig)
- $K_d$ = rated discharge coefficient (dimensionless) = $C_d$ = (actual flow rate) / (ideal flow rate)
- $K_w$ = correction factor for backpressure ( = 1 if discharging to atmosphere or if backpressure is less than 50% of inlet pressure)
- $K_c$ = rupture disc correction factor, if installed ( = 1 if none installed)
- $K_v$ = viscosity correction factor ( = 1 if Re > 10$^5$)
- 38 = all unit conversions compiled into one constant

Assuming no rupture disc is installed, no viscosity correction, and backpressure < 50% inlet pressure, the API 520 equation (using $C_d$ instead of $K_d$) boils down to:

$$ A = \frac{Q}{38C_d} \sqrt{\frac{SG}{dP}} $$

(4)

Rearranging Equation 4 yields:

$$ 38AC_d = Q \sqrt{\frac{SG}{dP}} $$

(5)

**Relationship Between $C_v$ and $C_d$**

The right hand side of Equation 5 is common with the flow coefficient equation, Equation 2 above. Therefore, for liquids:

$$ C_v = 38AC_d $$

(6)

A similar evaluation can be done for compressible gases and vapors (using Equation 11a in the ANSI/ISA 75.01.01 standard and Equation 3 in the API standard 520 Part 1, for example), but the relationship becomes:

$$ C_v = 27.66AC_d $$

(7)
The next question is: "Why are the constants different?" The answer is that the discharge coefficient for a given valve is smaller for a liquid than it is for a gas due to the expansion of the gas as it passes through the valve. For example, one manufacturer shows the discharge coefficient for one of their valves in liquid service is 0.579, but for gas service is 0.801. The ratio of the discharge coefficients is 0.801/0.579 = 1.38. The ratio of the constants in the above equations is 38/27.66 = 1.37, roughly equal.

Deriving the Numerical Constant in \( C_v \) to \( C_d \) Relationship

The next question a good engineer will ask is: "Where does the constant 38 come from?" The answer to that requires some unit analysis of the one-dimensional isentropic nozzle flow energy balance equation, which is given in Appendix B of the API 520 standard.

Using U.S. units for liquid, the mass flow rate per unit area through a nozzle (mass flux, \( G \)) using Equation B.1 and B.6 in the API standard, is:

\[
G = \frac{W}{a} = \sqrt{(2)(g)(144)p_dP}
\]

Where:

- \( G \) = mass flux in lb/sec-ft
- \( W \) = theoretical mass flow rate in lb/sec
- \( a \) = flow area in ft²
- \( g \) = 32.174 ft/sec²
- \( p \) = fluid density in lb/ft³
- \( dP \) = pressure drop across the relief, in lb/in²
- 144 = conversion between in² and ft²

Disregarding the unit conversion needed for the moment, the mass flow rate is related to the volumetric flow rate by:

\[
w = \rho Q
\]

Therefore, the mass flux is:

\[
G = \frac{w}{a} = \frac{\rho Q}{a} = \sqrt{(2)(g)(144)p_dP}
\]

Solving for area (\( a \)) and taking the fluid density into the square root:

\[
a = \frac{\rho Q}{\sqrt{(2)(g)(144)p_dP}} = \frac{Q\sqrt{\rho^2}}{\sqrt{(2)(g)(144)p_dP}} = Q \sqrt{\frac{\rho}{(2)(g)(144)p_dP}}
\]

The density (\( \rho \)) in Equation 11 is the fluid density, but the valve sizing equations use the specific gravity. Specific gravity is:

\[
SG = \frac{\rho_1}{\rho_0}
\]

Where \( \rho_1 \) = density of water at 60 °F = 62.37 lb/ft³.

\[
\rho_1 = (SG)(\rho_0) = \left( 62.37 \frac{lb}{ft^3} \right)(SG)
\]

Taking this relationship into the area equation yields:
\[
A = a \left( \frac{144 \text{ in}^2}{\text{ft}^2} \right) = \left( \frac{144 \text{ in}^2}{\text{ft}^2} \right) Q \sqrt{\frac{62.37 \text{ lb}}{\text{ft}^3} (SG)} \left( \frac{1}{2} \right) \left( \frac{144 \text{ in}^2}{\text{ft}^2} \right) dP
\]

Before we throw in all the units, we need the area in square inches, not square feet, so:

\[
A = a \left( \frac{144 \text{ in}^2}{\text{ft}^2} \right) = \left( \frac{144 \text{ in}^2}{\text{ft}^2} \right) Q \sqrt{\frac{62.37 \text{ lb}}{\text{ft}^3} (SG)} \left( \frac{1}{2} \right) \left( \frac{144 \text{ in}^2}{\text{ft}^2} \right) dP
\]

Now let's put in all the units:

\[
A = \frac{144 \text{ in}^2}{\text{ft}^2} \left( \frac{Q \text{ gal}}{\text{min}} \right) \left( \frac{\text{min}}{60 \text{ sec}} \right) \left( \frac{1 \text{ ft}^3}{7.48055 \text{ gal}} \right) \sqrt{\frac{62.37 \text{ lb}}{\text{ft}^3}} \left( \frac{1}{2} \right) \left( \frac{1 \text{ sec}^2}{32.174 \text{ ft}} \right) \left( \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right) \left( \frac{\text{in}^2}{\text{dPlb}} \right) \left( \frac{\text{SG}}{1} \right)
\]

\[
A = 0.02632Q \sqrt{\frac{SG}{dP}} = \frac{Q}{38} \sqrt{\frac{SG}{dP}}
\]

The discharge coefficient comes into the equation above because the flow rate, \( Q \), is the theoretical flow rate assuming incompressible isentropic flow. The discharge coefficient is the ratio of the actual flow to the theoretical flow:

\[
C_d = \frac{Q_{\text{actual}}}{Q_{\text{theoretical}}}
\]

Rearranged:

\[
Q_{\text{theoretical}} = \frac{Q_{\text{actual}}}{C_d}
\]

Substituting into the area equation above (Equation 16) yields the relief valve sizing equation:

\[
A = \frac{Q_{\text{actual}}}{38C_d} \sqrt{\frac{SG}{dP}}
\]

Summary

Over the course of history, the scientific and engineering study involving fluid flow in piping systems has resulted in developing different coefficients to characterize the hydraulic performance of various devices that obstruct fluid flow. Because engineers view the hydraulic performance of devices differently, mistakes can be made if the proper concepts and equations are not applied correctly. These can be costly mistakes in sizing and selecting the wrong equipment which can mean the difference between the system having sufficient pressure relieving capacity or the system rupturing during a high pressure relief incident.