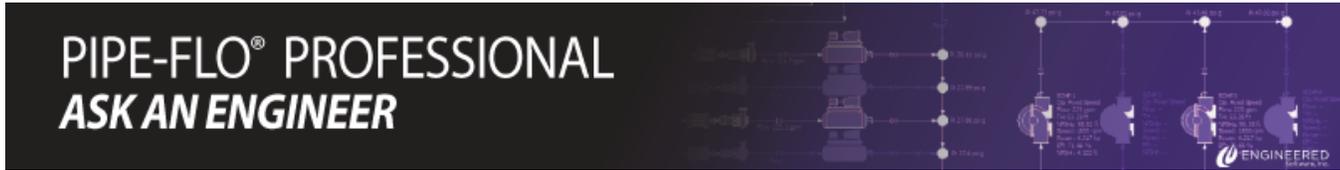


Relationship Between Pressure Drop and Flow Rate in a Pipeline



What is the relationship between pressure drop and flow rate in a pipeline?

by Jeff Sines, Senior Product Engineer at Engineered Software, Inc.

To understand the relationship between the pressure drop across a pipeline and the flow rate through that pipeline, we need to go back to one of the most important fundamental laws that governs the flow of fluid in a pipe: the Conservation of Energy, which for incompressible liquids, can be expressed using the Bernoulli Equation.

Total Fluid Energy

Daniel Bernoulli, a Swiss mathematician and physicist, theorized that the total energy of a fluid remains constant along a streamline assuming no work is done on or by the fluid and no heat is transferred into or out of the fluid. The total energy of the fluid is the sum of the energy the fluid possesses due to its elevation (elevation head), velocity (velocity head), and static pressure (pressure head).

$$\text{Total Energy} = \text{Elevation Head} + \text{Velocity Head} + \text{Pressure Head}$$

or:

$$TE = Z + \frac{v^2}{2g} + \frac{144P}{\rho}$$

where (in US Imperial Units):

- TE = total fluid energy (feet of fluid)
- Z = elevation of the fluid measured from a reference plane (feet)
- v = fluid velocity (feet/sec)
- g = gravitational constant (32.2 ft/sec²)
- P = static pressure (psi, or lb/in²)
- ρ = fluid density (lb/ft³)
- 144 = conversion from in² to ft²

Bernoulli Equation Applied Between Two Points

In reality, the flow of fluid between two points cannot be achieved without a loss of fluid energy due to friction and changes in momentum. The energy loss, or head loss, is seen as some heat lost from the fluid, vibration of the piping, or noise generated by the fluid flow. Head loss is a reduction in the capability of the fluid to do work and will act to reduce the static pressure of the fluid.

Between two points, the Bernoulli Equation can be expressed as:

$$Z_1 + \frac{v_1^2}{2g} + \frac{144P_1}{\rho} = Z_2 + \frac{v_2^2}{2g} + \frac{144P_2}{\rho} + H_L$$

where:

- 1 = upstream location
- 2 = downstream location
- H_L = head loss between the upstream and downstream locations (feet of fluid)

The Bernoulli Equation can now be re-arranged to show that the change in static pressure (dP = P₁ - P₂) is due to three components: a change in elevation, a change in velocity, and the energy lost to heat, noise, and vibration:

$$(P_1 - P_2) = \frac{\rho}{144} \left[(Z_2 - Z_1) + \left(\frac{v_2^2 - v_1^2}{2g} \right) + H_L \right]$$

Pressure Change due to Elevation Change

The change in elevation can either be positive or negative. In other words, the upstream location can be at a lower or higher elevation than the downstream location. If the fluid is flowing up to a higher elevation, this energy conversion will act to decrease the static pressure. If the fluid flows down to a lower elevation, the change in elevation head will act to increase the static pressure.

For example, evaluating just the change in elevation, consider 60 °F water flowing up a hill from an inlet elevation of 25 ft to an outlet elevation of 75 ft:

$$(P_1 - P_2) = \frac{\rho}{144} (Z_2 - Z_1) = \frac{62.4 \frac{lb}{ft^3}}{144 \frac{in^2}{ft^2}} (75 - 25ft) = 21.7 \text{ psi}$$

The positive value of dP indicates that the inlet pressure is greater than the outlet pressure, so pressure will decrease by 21.7 psi solely due to the change in elevation of the fluid as it flows uphill. Conversely, if the fluid is flowing down hill from an elevation of 75 ft to 25 ft, the result would be negative and there will be a 21.7 psi gain in pressure and the outlet pressure will be greater than the inlet pressure.

Pressure Change due to Velocity Change

Fluid velocity will change if the internal flow area changes. For example, if the pipe size is reduced, the velocity will increase and act to decrease the static pressure. If the flow area increases through an expansion or diffuser, the velocity will decrease and result in an increase in the static pressure. If the pipe diameter is constant, the velocity will be constant and there will be no change in pressure due to a change in velocity.

As an example, if an expansion fitting increases a 4 inch schedule 40 pipe to a 6 inch schedule 40 pipe, the inside diameter increases from 4.026" to 6.065". If the flow rate through the expansion is 368 gpm, the velocity goes from 9.27 ft/sec to 4.09 ft/sec. The change in static pressure across the expansion due to the change in velocity is:

$$(P_1 - P_2) = \frac{\rho}{144} \left(\frac{v_2^2 - v_1^2}{2g} \right) = \frac{62.4 \frac{lb}{ft^3}}{144 \frac{in^2}{ft^2}} \left(\frac{(4.09 \text{ ft/sec})^2 - (9.27 \text{ ft/sec})^2}{2(32.2 \text{ ft/sec}^2)} \right) = -0.466 \text{ psi}$$

The negative value of dP indicates that the outlet pressure will be greater than the inlet pressure. In other words, pressure has increased by almost 0.5 psi from inlet to outlet solely due to the conversion of velocity head to pressure head.

Pressure Change due to Head Loss

Since head loss is a reduction in the total energy of the fluid, it represents a reduction in the capability of the fluid to do work. Head loss does not reduce the fluid velocity (consider a constant diameter pipe with a constant mass flow rate), and it will not be effect the elevation head of the fluid (consider a horizontal pipe with no elevation change from inlet to outlet). Therefore, head loss will always act to reduce the pressure head, or static pressure, of the fluid.

There are several ways to calculate the amount of energy lost due to fluid flow through a pipe. The two most common methods are the Darcy-Weisbach equation and the Hazen-Williams equation. The root form of the Darcy equation can be found in the Crane TP-410, among other industrial references, and is:

$$H_L = f \frac{L}{D} \frac{v^2}{2g}$$

where:

- H_L = head loss (feet)
- f = Darcy friction factor (dimensionless)
- L = pipe length (feet)
- D = pipe inside diameter (feet)
- v = fluid velocity (ft/sec)
- g = gravitational constant (32.2 ft/sec²)

The Darcy friction factor, f , takes into account the pipe roughness, diameter, fluid viscosity, density, and velocity by first calculating the Reynolds Number and Relative Roughness. Once calculated, the head loss can then be converted to a change in pressure using:

$$dP = \frac{\rho H_L}{144}$$

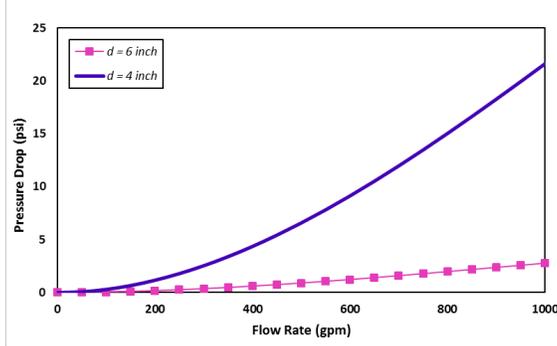
There are other forms of the Darcy equation in the Crane TP-410 that use variables with different units and a numerical constant that combines all the unit conversions together. For example:

$$dP = 2.161 * 10^{-4} \left(\frac{fL\rho Q^2}{d^5} \right)$$

where:

- Q = flow rate (gpm)
- d = pipe diameter (inches)

The graph below shows the resulting pressure drop for water at 60 F over a range of flow rates for a 100 foot long pipe for both 4 inch and 6 inch schedule 40 piping.



Summary

To determine the total change in the static pressure of a fluid as it flows along a pipeline, all three components of the Bernoulli Equation must be considered individually and added together. A change in elevation may cause the pressure to decrease, a change in velocity may cause it to increase, and the head loss may cause it to decrease. The net effect will depend on the relative magnitudes of each change.

It is possible that the static pressure of the fluid actually increases from inlet to outlet if the change in elevation or velocity results in an increase of pressure greater than the decrease that results due to head loss.

The old saying that "fluid always flows from high pressure to low pressure" is not quite accurate. The more accurate way to state this is that "fluid always flows from a region of higher total energy to a region of lower total energy".

References: [Crane Technical Paper 410](#) Copyrighted 2009 CRANE Co.